Dimensional inflation of spacetime into a $4D$ mirrored universe

Wanpeng Tan*

Department of Physics and Astronomy,

Institute for Structure and Nuclear Astrophysics (ISNAP),

University of Notre Dame, Notre Dame, Indiana 46556, USA

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Mirror symmetry, understood as local orientation symmetry, has been conjectured to allow two sectors of particles in our $4D$ universe. This idea is further developed by considering dimensional transitions of spacetime during the inflating early universe. Based on mathematical results from string theory, supersymmetric mirror models are constructed for spacetime of different dimensions to understood the early universe and black hole physics. Under $4D$, dark matter is just "mirror matter" while dark energy originates in the nearly-canceled residual out of two gauge vacuum energies of the two sectors. The new framework is particularly intriguing as laboratory experiments are ready to test its unique prediction of neutral particle oscillations.

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I. Introduction.—The existence of a mirror sector of the Universe has been conjectured since Lee and Yang published their Nobel Prize-winning work on parity violation [1] that was later experimentally verified by Wu's group [2]. In the 1960s, Wigner first realized, when lecturing at a summer school, that the discrete symmetries of the Lorentz group could double the number of known particle states [3] (which was also discussed further by Weinberg in his classic textbook on QFT [4]). Motivated by parity violation and later revealed CP violation, three Soviet Union scientists, Kobzarev, Okun, and Pomeranchuk, proposed the concept of mirror symmetry — it is conceivable that there exist two sectors of particles sharing the same gravity but governed by two separate gauge groups under $4D$ spacetime [5].

After some silence, the idea of mirror matter was revived in the 1980s mainly from interesting perspectives in astrophysics and cosmology [6–8]. Later attempts to introduce ad hoc feeble interactions between the two sectors might have been too conservative [9–11]. Most

* wtan@nd.edu

TABLE I. Central charge contributions from Faddeev-Popov ghosts, resulting critical dimensions, and corresponding supersymmetric mirror models are listed.

ghost types	$(\lambda, 1-\lambda)$	c_g	c (all ghosts)	critical D	models
commuting	$(1/2, 1/2)$	-1	-3	2	SMM2 / SMM2b
anti-commuting	$(1, 0)$	-2			
commuting	$(3/2, -1/2)$	11	-15	10	SMM4 / SMM4b
anti-commuting	$(2, -1)$	-26			

recent works [12–23] with a new understanding of mirror symmetry, supersymmetry (SUSY), and dimensional phase transitions of spacetime could potentially solve a variety of puzzles in fundamental physics and cosmology and may indeed lead us to new physics beyond the Standard Model we have been looking for. Furthermore, a review on the phenomenological mirror-ordinary particle oscillations of the new theory and its laboratory tests can be seen in Ref. [24]. The deep meanings and profound consequences of mirror symmetry, in particular, in connection to string theory, are discussed in Ref. [25].

In the following we will primarily focus on understanding the mathematical constructs in string theory and connecting these mathematical results to physical meanings in mirror matter theory. In doing so, we may need to significantly alter the original understanding in string theory and reinterpret it within the context of the new framework. With the assistance of powerful results from string theory and mirror symmetry, we will present the new picture of our universe undergoing dimensional transitions of spacetime, naturally resolving puzzles such as dark energy and dark matter.

II. Dimensional phase transitions of spacetime via inflation.—One of the most astonishing achievements in superstring theory is that a critical dimension of $D = 10$ is required for the target spacetime to be consistent. However, there exists, though not well advertised, another critical dimension of $D = 2$ in superstring theory as demonstrated below. Under the BRST formalism, it can be shown that the total central charge of the Virasoro algebra in string theory must be zero. On the one hand, superspace (X^D, θ^D) contributes a central charge of $c = D + D/2 = 3D/2$. On the other hand, the Faddeev-Popov ghosts, with conformal weights of $(\lambda, 1-\lambda)$, contribute a canceling central charge [26],

$$c_g = -2(-1)^{2\lambda}(6\lambda^2 - 6\lambda + 1). \tag{1}$$

As shown in Table I, two critical dimensions ($D = 2, 10$) emerge under two different

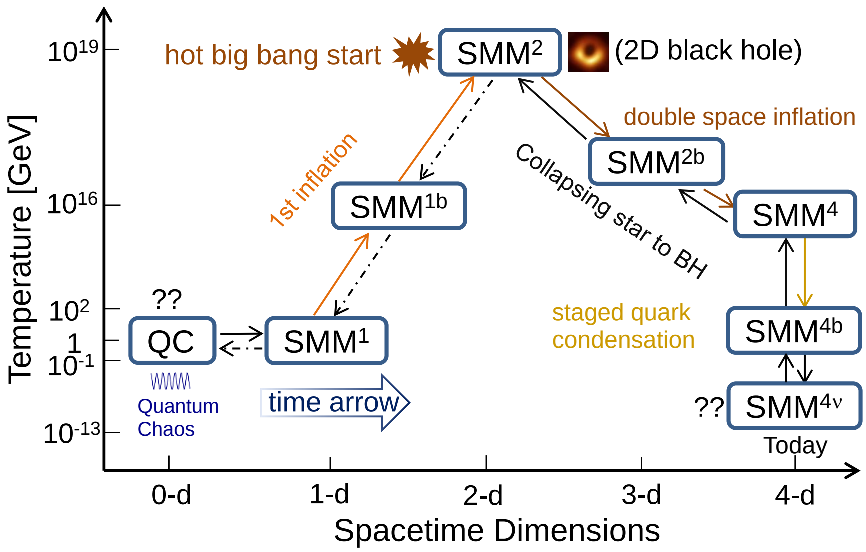

FIG. 1. The schematic diagram (not to scale) is shown for the Supersymmetric Mirror Models at various phases of spacetime. Adapted from Ref. [22].

scenarios. Specifically, as discussed below, $D = 2$ with ghosts of spin $1/2$ and 1 corresponds to the supersymmetric mirror models of SMM2 and SMM2b, while the critical dimension of $D = 10$ with ghosts of spin $3/2$ and 2 leads to the models of SMM4 and SMM4b.

In the framework of the new mirror theory, extended spacetime can undergo dimensional transitions, leading to the emergence of different sets of particles and interactions at different dimensions of the extended spacetime [19, 22, 25]. These transitions may occur during cosmic inflation of the early Universe and during the collapse of a massive star into a black hole at late stages of stellar evolution. Mirror symmetry [25], understood as local orientation symmetry connecting two sectors of particles in quantum field theory (QFT), are closely related to T-duality [27] and Calabi-Yau mirror symmetry [28] in string theory.

Modern quantum field theories can be formulated under the mathematical language of fiber bundle theory [29]. The critical dimension of the target space in string theory can then be understood as the dimension of the base manifold of a fiber bundle. Fig. 1 shows the dimensional evolution of the Universe involving the supersymmetric mirror models discussed below. In particular, the phase transitions result from spontaneous symmetry breaking of mirror symmetry at different dimensions of spacetime.

III. Low-dimensional supersymmetric mirror models.—Under the scenario of dimensional evolution or inflation of spacetime, the mirror models (SMM1 and SMM1b) for the beginning of the Universe have Lagrangians consisting of a single real scalar φ [19],

$$\mathcal{L}_{\text{SMM1}} = \frac{1}{2}\dot{\varphi}^2, \quad \mathcal{L}_{\text{SMM1b}} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi^2) \tag{2}$$

which, under $1D$ (space)time, are fairly trivial with the mirror or orientation symmetry being identified as its holonomy group $O(1)$ or time reversal symmetry [19, 22]. They describe $1D$ cases before supersymmetry or string theory becomes applicable. Starting from $2D$, the true essence of string theory, i.e., the meaning of strings, is its natural requirement of complex structures in internal spaces of QFT.

In general, fiber spaces and/or other compactified spaces provide holonomy gauge groups for representations of gauge interactions. Scalars emerge from the condensation of fermions of opposite chiralities, which may lead to a massive world or further dimensional transitions. Self-consistent gauge and chiral supermultiplets appear naturally and become building blocks for the supersymmetric mirror models.

In particular, under the critical dimension of $D = 2$, a gauge supermultiplet and a chiral supermultiplet emerge and can be used to build the models of SMM2 and SMM2b. Their Euclidean actions are proposed as follows,

$$\mathcal{S}^E_{\text{SMM2}} = \int d^2z\, \{\frac{1}{4}(\bar{\partial}A(z) - \partial\bar{A}(\bar{z}))^2 + \lambda(z)\bar{\partial}\lambda(z) + \bar{\lambda}(\bar{z})\partial\bar{\lambda}(\bar{z})\} \tag{3}$$

which contains a gauge supermultiplet of massless free Majorana fermions $\lambda, \bar{\lambda}$ and $U(1)$ bosons $A, \bar{A}$ in conformal gauge, similar to a string theory model on a $2D$ worldsheet, and

$$\mathcal{S}^E_{\text{SMM2b}} = \int d^2z\, \{\partial\phi\bar{\partial}\phi - \lambda\bar{\partial}\lambda - \bar{\lambda}\partial\bar{\lambda} - V''(\phi)\lambda\bar{\lambda} - \frac{1}{4}(V'(\phi))^2\} \tag{4}$$

which is a $N = (1, 1)$ supersymmetric model where $\phi = \phi_L(z) + \phi_R(\bar{z})$ is the sum of both holomorphic and anti-holomorphic scalars from condensation of Majorana fermions in Eq. 3. The terms in these actions involving auxiliary fields to close the supersymmetry algebra off-shell are omitted for simplicity. The symmetric Lagrangians between holomorphic and anti-holomorphic modes exactly demonstrate the mirror symmetry in $2D$ cases.

SMM2 has been applied to well describe the microphysics, and their entropy and temperature, of Schwarzschild black holes as $2D$ boundaries of $4D$ spacetime [23]. SMM2b could be used to explain the dynamics of cosmic inflation or black-hole collapsing processes. Strings in $2D$ string theory, or equivalently complex structures in the description of free Majorana fermions with $U(1)$ gauge under the new framework , could also be the origin of string or complex structures in higher dimensions extended from further spatial inflation.

IV. Splitting of spaces and fermions in $4D$ spacetime.—For higher dimensional supersymmetric mirror models (SMM4 and SMM4b), the base manifold with critical dimension $D = 10$ is broken into two parts: a $4D$ inflated spacetime and a $6D$ compactified Calabi-Yau space. The related string theory model is the so-called heterotic model [30]. However, instead of selecting one chiral model, two chirally symmetric copies of heterotic strings should be combined to be consistent with the requirements of mirror symmetry. That is, a $D = 26$ left-moving bosonic string combined with a $D = 10$ right-moving susperstring provides the ordinary sector, while a right-moving bosonic string plus a left-moving superstring of the same dimensions gives rise to the mirror sector, i.e.,

$$(\textbf{Heterotic String})_{\textbf{Left}} + (\textbf{Heterotic String})_{\textbf{Right}} \Rightarrow \textbf{SMM4/SMM4b}. \qquad (5)$$

The reason why only four dimensions can be fully extended in the base manifold could be understood in the simple ϕ^4 renormalization group theory (RG). Under $4D$ spacetime, RG calculations show that the ϕ^4 term is marginal and becomes irrelevant in higher dimensions. This means that only free scalar fields can exist in $D > 4$ quantum field theory, making the Higgs mechanism and inflation impossible in higher dimensions. Therefore, our extended spacetime cannot exceed four dimensions for finite or renormalizable models of massive fields. The $6D$ Calabi-Yau space out of the $10D$ base manifold must therefore be compactified for consistency. As a result, all associated spaces, including the tangent/cotangent spaces and the extra 16 dimensions of each chiral bosonic string as a compactified fiber space, must be split into two parts as the base manifold does.

Baez demonstrated that the Standard Model gauge group G_{SM} is the holonomy group of a $10D$ Calabi-Yau manifold whose tangent spaces split into orthogonal $4D$ and $6D$ subspaces, each preserved by the complex structure and parallel transport [31]. It uses the following isomorphism that the Georgi–Glashow $SU(5)$ grand unified theory [32] also relies on,

$$G_{SM} = S(U(2) \times U(3)) = SU(3) \times SU(2) \times U(1)/Z_6 \qquad (6)$$

which amazingly gives the representation of one generation of fermions in the Standard Model (see details in Ref. [31] and Table II). Note that $SU(2)$ here does not explain the

TABLE II. Fundamental representation of ordinary fermions for critical dimension $D = 10$ with $4D/6D$ splitting exactly overlaps with one generation of fermions in the Standard Model [31]. Lepton and quark space singlets are labeled with $s^2 = \wedge^0\mathbb{C}^2$ and $s^3 = \wedge^0\mathbb{C}^3$ and the corresponding vector reps are $v^2 = \wedge^1\mathbb{C}^2$ and $v^3 = \wedge^1\mathbb{C}^3$. Note that $v^2 = v^{2*}$ and Hodge star operator $*$ maps to the dual or anti-particle rep.

exterior algebra	decomposition	SM fermions	$SU(3)\times SU(2)\times U(1)$ rep	$SU(5)$ rep
$\wedge^0(\mathbb{C}^3\oplus\mathbb{C}^2)$	$s^3\otimes s^2$	$\bar{\nu}_R$	$(\mathbf{1},\mathbf{1},\mathbf{0})$	
$\wedge^1(\mathbb{C}^3\oplus\mathbb{C}^2)$	$v^3\otimes s^2$	$d_R^{r,g,b}$	$(\mathbf{3},\mathbf{1},-\mathbf{1/3})$	$\mathbf{5}$
	$\oplus\ s^3\otimes v^2$	$\bar{e}_L,\bar{\nu}_L$	$(\mathbf{1},\mathbf{2},\mathbf{1/2})$	
$\wedge^2(\mathbb{C}^3\oplus\mathbb{C}^2)$	$v^{3*}\otimes s^2$	$\bar{u}_R^{r,g,b}$	$(\bar{\mathbf{3}},\mathbf{1},-\mathbf{2/3})$	$\mathbf{10}$
	$\oplus\ v^3\otimes v^2$	$d_L^{r,g,b}, u_L^{r,g,b}$	$(\mathbf{3},\mathbf{2},\mathbf{1/6})$	
	$\oplus\ s^3\otimes s^{2*}$	$\bar{e}_R$	$(\mathbf{1},\bar{\mathbf{1}},\mathbf{1})$	
$\wedge^3(\mathbb{C}^3\oplus\mathbb{C}^2)$	$v^3\otimes s^{2*}$	$u_R^{r,g,b}$	$(\mathbf{3},\bar{\mathbf{1}},\mathbf{2/3})$	$\overline{\mathbf{10}}$
	$\oplus\ (v^3\otimes v^2)^*$	$\bar{d}_L^{r,g,b}, \bar{u}_L^{r,g,b}$	$(\bar{\mathbf{3}},\bar{\mathbf{2}},-\mathbf{1/6})$	
	$\oplus\ s^{3*}\otimes s^2$	e_R	$(\bar{\mathbf{1}},\mathbf{1},-\mathbf{1})$	
$\wedge^4(\mathbb{C}^3\oplus\mathbb{C}^2)$	$(v^3\otimes s^2)^*$	$\bar{d}_R^{r,g,b}$	$(\bar{\mathbf{3}},\bar{\mathbf{1}},\mathbf{1/3})$	$\bar{\mathbf{5}}$
	$\oplus\ (s^3\otimes v^2)^*$	e_L,ν_L	$(\bar{\mathbf{1}},\bar{\mathbf{2}},-\mathbf{1/2})$	
$\wedge^5(\mathbb{C}^3\oplus\mathbb{C}^2)$	$(s^3\otimes s^2)^*$	ν_R	$(\bar{\mathbf{1}},\bar{\mathbf{1}},\mathbf{0})$	

chiral feature of the weak $SU_L(2)$ group in the Standard Model.

Baez's approach is very effective in building up representations of fermions according to the splitting of the base manifold, which is naturally associated with complex tangent/ cotangent spaces. The exterior algebra of such a complex (holomorphic) cotangent space splitting $\mathbb{C}^5 \to \mathbb{C}^3 \oplus \mathbb{C}^2$ then gives a fundamental representation of ordinary fermions as shown in Table II. Meanwhile, the splitting of the complex conjugate (anti-holomorphic) cotangent space gives a similar exterior algebra representation $\wedge(\bar{\mathbb{C}}^3\oplus\bar{\mathbb{C}}^2)$ for mirror fermions.

The defining representation, $\mathbf{5} = (d_R^{r,g,b}, \bar{e}_L, \bar{\nu}_L)$ of $SU(5)$ in Table II, shows why quarks are defined in the $6D$ CY space while leptons are born in $4D$ spacetime. This explains why quarks have color confinement while leptons can propagate individually in spacetime. There are a total of 32 degrees of freedom (DoFs) in each sector of fermions, but only 30 of them participate in gauge interactions within their sector. The two singlets ν_R and $\bar{\nu}_R$ participate in the $SU_R(2)$ interactions of the mirror sector while ν_L and $\bar{\nu}_L$ are gauge singlets in the mirror sector. Therefore, we say that the two sectors share the same set of neutrinos.

V. Gauge groups and anomalies.—The group $SU(3)\times SU(2)\times U(1)$ obtained from the splitting of the base manifold discussed above is not exactly the desired gauge group. In the following, we will adapt Baez's approach also for fiber spaces in order to obtain the true gauge groups, in particular, the chiral weak $SU(2)$ group, in $4D$ spacetime.

The two heterotic strings provide not only a matched $10D$ base space with two sets of fermions but also two $16D$ unmatched chiral spaces to be compactified. For the ordinary sector, the 16 extra dimensions of the left-moving bosonic string have to be compactified into a fiber space with $SU_L(8)$ holonomy that has to be split with respect to the base manifold via Baez's approach,

$$SU_L(8) \longrightarrow SU_L(6) \times SU_L(2) \times U_L(1)/Z_6 \tag{7}$$

where one of the subspaces has to be a $4D$ fiber space of extended spacetime giving the group $SU_L(2)$ and the CY quark space then has to take charge of the group $SU_L(6)$. We will see that the chiral nature of these fiber groups is critical for establishing the new framework.

We can immediately identify $SU_L(2)$ as the weak gauge group of the Standard Model for the ordinary sector. However, the flavor group $SU_L(6)$ and chiral $U_L(1)$ cannot be gauged due to anomalies. We can identify $U_L(1)$ charge as the difference $B-L$ between the baryon number B and the lepton number L [25], after chiral breaking, leading to a global symmetry $U_V^{B-L}(1)$ for $B-L$ conservation.

The flavor group $SU_L(6)$ has two ways to break down. One is for it to be further broken to a gaugeable isospin symmetry $SU_I(2)$ for quarks as all $SU(2)$ groups are automatically anomaly-free. So in this case, the complete gauge group for the ordinary sector becomes,

$$G_{\rm SMM4} = U_Y(1) \times SU_L(2) \times SU_c(3) \times SU_I(2) \tag{8}$$

which provides massless gauge bosons with DoFs of $n_b = 30$. This coincides with the DoFs of one generation SM fermions $n_f = 30$ without counting ν_R and $\bar{\nu}_R$ as shown in Table II where it indeed shows that ν_R and $\bar{\nu}_R$ do not participate in any ordinary gauge interactions. Thus, we obtain an ordinary gauge supermultiplet of $n_b = n_f = 30$ involving one generation SM particles and a similar supermultiplet in the mirror sector.

On the other hand, $SU_L(6)$ could also be left as a completely global symmetry for quark flavors, explaining the existence of six flavors or three generations of quarks in the Standard Model. Meanwhile, the existence of three generations of leptons can be understood as follows: when we complexify (co)tangent spaces of $4D$ spacetime, there are three ways to pair the time dimension with one of three spatial dimensions, resulting in three different (co)tangent spaces that represent three generations of leptons.

Then the ordinary gauge group in this case is exactly the well known SM group,

$$G_{\text{SMM4b}} = U_Y(1) \times SU_L(2) \times SU_c(3) \tag{9}$$

where $n_b(\text{SM}) = 27$ after spontaneous symmetry breaking gives masses to $W^{\pm}$ and Z^0 bosons of $SU_L(2)$. Meanwhile, the global flavor group $SU_L(6)$ for quarks breaks down as follows,

$$SU_L(6) \xrightarrow{\text{chiral breaking}} SU_V(2) \times U_V^t(1) \times U_V^b(1) \times U_V^c(1) \times U_V^s(1) \tag{10}$$

where $SU_V(2)$ and $U_V^{t,b,c,s}(1)$ are a set of leftover global symmetries that conserve isospin, baryon number, and t, b, c, s numbers of quarks. The chiral $U_A(1)$ breaking from $U_L(1)$ produces no pseudo-Nambu-Goldstone boson (pNGB) as it is dynamically canceled by other flavor $U_A(1)$'s of t, b, c, s quarks produced in Eq. 10. As a result, the well-known $U_A(1)$ problem is resolved without the need for the hypothetical axion. In the end, the flavor $SU_L(6)$ breaking produces pNGBs with DoFs of 63. Combined with the DoFs of gauge bosons, we have a total of $n_b = 90$. This gives us a pseudo-SUSY multiplet with $n_b = n_f = 90$ for three generations of SM particles. More details on this aspect can be seen in Refs. [15, 18].

Similarly, the breakdown of $SU_R(8)$ from the right-handed heterotic string gives us the gauge groups in the mirror sector, similar to Eqs. 8-9.

VI. Supersymmetric mirror models in $4D$ spacetime.—At the critical dimension of $D = 10$, there are two possible choices of supermultiplets. For the case of one generation SM particles in the UV limit, SMM4 must be massless, and therefore we only need massless gauge supermultiplets for its on-shell Lagrangian,

$$\mathcal{L}_{\text{SMM4}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + i\bar{\psi}_j \gamma^\mu D_\mu \psi_j - \frac{1}{4} G'^a_{\mu\nu} G'^{a\mu\nu} + i\bar{\psi}'_j \gamma^\mu D'_\mu \psi'_j \tag{11}$$

where for the ordinary sector, $G^a_{\mu\nu}$ $(a = 1, 2, ..., 15)$ is the gauge field strength tensor and the gauge covariant derivative $D_\mu = \partial_\mu - igT^a A^a_\mu$ depends on gauge symmetry generators T^a and gauge bosons A^a_μ as given in Eq. 8. The 15 massless Dirac fermion fields ψ_j represent exactly one generation of quarks and leptons (excluding ν_R and $\bar{\nu}_R$) as shown in Table II.

The reason why only one generation of particles is present is because spacetime is still in the early stages of dimensional transition from $2D$ to $4D$, i.e., the two new spatial dimensions have not yet been fully inflated. Therefore, there is only one way to complexify the (co)tangent spaces of this incomplete $4D$ spacetime by pairing the two original space and time dimensions together into one complex dimension. This single type of complexified (co)tangent spaces then represents one generation of leptons. Meanwhile, $SU(6)$ is not global yet, so quarks also have one generation.

The mirror sector has similar terms in the Lagrangian with the mirror transformation applied as $\mathcal{M}: \psi_L \to -\psi'_L,\ \psi_R \to \psi'_R,\ A_\mu \to A'_\mu$. Since $SU(2)$ does not preserve two different internal orientations, neutrinos are shared between the two sectors, leading to the following neutrino degeneracy relations,

$$\nu_L = -\nu'_L \text{ and } \nu_R = \nu'_R. \tag{12}$$

Massive chiral supermultiplets arise due to the emergence of three generations of particles and the breakdown of the global flavor $SU(6)$ group. At lower energies, six flavors of quarks can be condensed into six Higgs-like scalars, $H_{u,d,c,s,t,b}$, which must participate in weak $SU_L(2)$ interactions as a doublet $H = (\phi^+, \phi)$ since each condensate must contain one left-handed quark. After undergoing spontaneous symmetry breaking and giving masses to other particles, these scalars will have a single DoF left for each and become simple real scalars, $\phi_{u,d,c,s,t,b}$, with respect to the new vacuum configuration. Along with three families of neutrino singlets, $\nu_R^{1,2,3}$ and $\bar{\nu}_R^{1,2,3}$, they form three chiral supermultiplets for the ordinary sector. The mirror sector has similar supermultiplets.

Then we can obtain the low energy model SMM4b as an extension to the Standard Model,

$$\mathcal{L}_{\text{SMM4b}} = \mathcal{L}_{\text{SM}}(A_\mu, \psi_L, \psi_R, H) + \mathcal{L}'_{\text{SM}}(A'_\mu, -\psi'_L, \psi'_R, -H') \tag{13}$$

which includes pseudo-SUSY multiplets of gauge bosons A_μ and A'_μ, and Dirac fermions ψ and ψ' of three generations. The model also includes three sets of chiral supermultiplets involving six scalars (H,H') for each sector. The mirror symmetry observed in this model can be demonstrated by,

$$\mathcal{M}: \psi_L \to -\psi'_L,\ \psi_R \to \psi'_R,\ A_\mu \to A'_\mu,\ H \to -H'. \tag{14}$$

The Lagrangian for each sector in the SMM4b model is largely identical to that of the Standard Model, with two exceptions. The Higgs mechanism is the same, but there are six

Higgs scalars in each sector of SMM4b. Early calculations showed that six Higgs particles can indeed account for the unification of running coupling constants toward the UV limit [33]. The other exception is that the Yukawa mass terms of the Dirac neutrinos shared between the two ordinary and mirror sectors in SMM4b can be obtained as follows [18],

$$-y(\bar{\nu}_L \nu_R \phi + \bar{\nu}'_L \nu'_R \phi' + h.c.) = -y(\bar{\nu}_L \nu_R (\phi - \phi') + h.c.) \tag{15}$$

which take into account the neutrino degeneracy conditions in Eq. 12. The masses of the neutrinos are then determined by the ordinary-mirror mass splitting scale of $\langle \phi - \phi' \rangle \sim v - v' = \delta v$ with a fairly well constrained relative scale of $\delta v / v = 10^{-15}$–10^{-14} [12], which agrees very well with current experimental constraints on neutrino masses [18].

VII. Dark energy and Higgs mechanism.—With mirror symmetry, we understand that the ordinary sector is left-moving or holomorphic while the mirror sector is right-moving or anti-holomorphic. There are no gauge interactions connecting the two sectors, meaning that any field from the two sectors must be harmonic, i.e., the sum of a holomorphic function and an anti-holomorphic function. This means that the ordinary gauge vacuum $|0\rangle$ and the mirror gauge vacuum $|\bar{0}\rangle$ have to satisfy the following conditions,

$$\chi(z)\,|\bar{0}\rangle = 0, \quad \chi(\bar{z})\,|0\rangle = 0 \tag{16}$$

for any ordinary (holomorphic) field $\chi(z)$ and mirror (anti-holomorphic) field $\chi(\bar{z})$.

When we take the vacuum expectation value of any operator or correlation functions, these two vacuum states naturally annihilate any possible product mixing of ordinary and mirror fields, effectively breaking the SMM4b Lagrangian into two completely separate parts as shown in Eq. 13. This holds even for Yukawa mass and Higgs potential terms. Condensation from ordinary/mirror quarks of opposite chiralities then gives an effective total Higgs field for both sectors as a simple sum, $\Phi = \phi + \phi'$, which is obviously harmonic and satisfies $\langle 0|\Phi|0\rangle = \langle \phi \rangle$ and $\langle \bar{0}|\Phi|\bar{0}\rangle = \langle \phi' \rangle$ that would cleanly separate the Yukawa mass and Higgs potential terms for the two sectors as expected.

Now we can examine the interesting quartic Higgs term that should contribute to the vacuum energy as,

$$\rho_{\text{vac}} \sim \lambda \left\langle \Phi \right\rangle^4 = \lambda \left\langle \phi + \phi' \right\rangle^4 \tag{17}$$

which, under any of the above gauge vacua, will simply reduce to the expectation value of the normal Higgs term within the corresponding sector. However, the gravitational vacuum for general relativity is probably better defined as a split-complex or hyperbolic structure of the two gauge vacua,

$$|g\rangle = |0\rangle + j\,|\bar{0}\rangle\,, \quad \langle g| = \langle 0| - j\,\langle \bar{0}| \tag{18}$$

where $j^2 = +1$. If we evaluate the quartic term under the gravitational vacuum $|g\rangle$, we obtain

$$\rho_{\rm vac} \sim \lambda\,\langle g|\Phi|g\rangle^4 = \lambda(\langle 0|\phi|0\rangle - \langle\bar{0}|\phi'|\bar{0}\rangle)^4 = \lambda(v-v')^4 \sim (10^{-3}\,\mathrm{eV})^4 \tag{19}$$

which is amazingly consistent with observed dark energy density [18].

In comparison with Eq. 15, it is clear that the dark energy scale and neutrino masses share the same origin from spontaneous mirror symmetry breaking of the two sectors. Dark energy is just the residual effect of spontaneous mirror symmetry breaking and so are neutrino masses. The same model parameters have also quantitatively and consistently explained many other puzzles including dark matter [12] and matter-antimatter imbalance of the Universe [15].

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