

Supplemental Material for “Mirror matter and the self-duality of $S(U(2) \times U(3))$ ”: portal basis

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PORTAL BASIS AND SELECTION RULES

This Supplemental Material lists the cross-sector portal basis behind the counts of Sec. V C: each family of gauge-invariant operators $O_1 O_2$ built from a G_{SM} -singlet block of sector 1 and a G'_{SM} -singlet block of sector 2, with its field content, Lorentz structure and number of real parameters, and the parameters that survive each mirror element. Derivative operators are reduced by integration by parts, the equations of motion and the Bianchi identity (Warsaw-type on-shell basis).

Six-quark structures. In each sector, with the chiral fields of the sector ($u \equiv u_R$, $d \equiv d_R$, $Q \equiv Q_L$ in sector 1; with mirror content u'_L , d'_L , Q'_R in sector 2), define the three-quark spinors $N_{udd} \equiv (udd) = \epsilon_{abc}(u_a^T C d_b) d_c$ and $N_{QQd} \equiv (QQd) = \epsilon_{abc}\epsilon_{ij}(Q_{ai}^T C Q_{bj}) d_c$ (a, b, c color and i, j isospin indices, $\epsilon_{123} = \epsilon_{12} = 1$, $C = i\gamma^0\gamma^2$), and $N'_{udd} \equiv (u'd'd')$, $N'_{QQd} \equiv (Q'Q'd')$ in sector 2, with all three fields primed. N_{udd} and N_{QQd} are right-handed; N' is left-handed with mirror content and right-handed with copy content. The operators (XY') , $X, Y \in \{udd, QQd\}$, are $\bar{N}_X N'_Y$ with mirror content and $N_X^T C N'_Y$ with copy content (the other pairing vanishes by chirality); $[SSS]$ is the product of three cross-sector scalar bilinears, $\mathcal{O}_{[SSS]} = \epsilon_{abc}\epsilon_{a'b'c'}(\bar{u}'_a u_a)(\bar{d}'_b d_b)(\bar{d}'_c d_c)$ with mirror content and $\epsilon_{abc}\epsilon_{a'b'c'}(u_a^T C u'_{a'})(d_b^T C d'_{b'})(d_c^T C d'_{c'})$ with copy content. The four (XY') span 8 real dimensions, $[SSS]$ raises this to 10 (it carries the part of $(udd) \otimes (u'd'd')$ that $\bar{N}N'$ does not), and the tensor form with $\sigma^{\mu\nu} \otimes \sigma_{\mu\nu}$ on the first two bilinears adds nothing (rank 10). With $\mathcal{L} \supset \sum c \mathcal{O} + \text{h.c.}$ the family has 5 complex structures, 10 real coefficients; with mirror content every one has $\Delta B = -\Delta B' = \pm 1$. Under $\mathcal{M}_P$: $c_{(udd)(u'd'd')} \in \mathbb{R}$, $c_{(QQd)(u'd'd')} = c_{(udd)(Q'Q'd')}^*$, $c_{(QQd)(Q'Q'd')} \in \mathbb{R}$, $c_{(udd)(u'd'd')}^{[SSS]} \in \mathbb{R}$ (5/10 survive); under $\tilde{\mathcal{M}}$ (copy content): $c_{(udd)(u'd'd')}$ free, $c_{(QQd)(u'd'd')} = c_{(udd)(Q'Q'd')}^*$, $c_{(QQd)(Q'Q'd')} \in \mathbb{R}$, $c_{(udd)(u'd'd')}^{[SSS]}$ free (8/10).

Tables II–X list the operators of each family. Operator-by-operator images under the mirror elements (frame-D forms as in the family table). An entry $\pm O_j$ ($\pm O_j^\dagger$) in row k means $X(O_k) = \pm O_j$ ($\pm O_j^\dagger$). With $\mathcal{L} \supset \sum_k c_k O_k$ (+h.c. for complex O_k , “real” = 2), invariance requires $c_j = \pm c_k$ ($c_j = \pm c_k^*$); a self-image $-O_k$ sets $c_k = 0$, and a self-image $+O_k^\dagger$ fixes the phase (c_k real). Fermionic families are n/a under $\mathcal{M}$ and $\tilde{\mathcal{M}}_P$. Six-quark notation as in the six-quark definitions.

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TABLE I. Families of cross-sector portals and their survivors.

Portal basis of $G_{SM} \times G'_{SM}$ (one generation per sector; two down-type flavors for $\Delta S = 2$ only), on shell. Columns: real parameters surviving each element, as survivors/total, in the frame where all matter is electric: $\mathcal{M}_P \rightarrow \text{swap}(P \otimes P)$, $\tilde{\mathcal{M}} \rightarrow \text{swap}$, $\mathcal{M} \rightarrow \text{swap}(C \otimes 1)$, $\tilde{\mathcal{M}}_P \rightarrow \text{swap}(CP \otimes P)$ (frame D; mirror content for $\mathcal{M}_P$, copy content for $\tilde{\mathcal{M}}$; the frame-D' forms give the same pattern). n/a: $\mathcal{M}$ and $\tilde{\mathcal{M}}_P$ are not symmetries of chiral matter. Sector-1 fields $Q_L, u_R, d_R, L_L, e_R, H$; sector 2 with mirror content $Q'_R, u'_L, d'_L, L'_R, e'_L, H'$, with copy content the same chiralities as sector 1. $J_H^\mu J_{H\mu}'$ is in the dimension-6 bosonic family; the current-current family couples a current of one sector (fermion or Higgs) to a fermion current of the other. Dimension-6 total: 85 real parameters.

family	operators (sector 1 $\times$ sector 2)	Lorentz structure	dim.	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$	$\mathcal{M}$	$\tilde{\mathcal{M}}_P$
dim.-4 bosonic	$ H ^2 H' ^2, B_{\mu\nu}B'^{\mu\nu} (\epsilon_D),$ $B_{\mu\nu}\tilde{B}'^{\mu\nu} (\theta_{\text{mix,D}})$	bosonic, Hermitian	4	3	2/3; forbids $\theta_{\text{mix,D}}$	3/3	1/3; forbids $\epsilon_D,$ $\theta_{\text{mix,D}}$	2/3; forbids ϵ_D
dim.-6 bosonic	$ H ^4 H' ^2, H ^2 H' ^4;$ $\partial_\mu H ^2\partial^\mu H' ^2; J_H^\mu J_{H\mu}';$ $ H ^2X'X', H ^2X'\tilde{X}',$ $ H' ^2XX, H' ^2X\tilde{X}$ ($X = B, W, G$); $ H ^2BB',$ $ H ^2B\tilde{B}', H' ^2BB',$ $ H' ^2B\tilde{B}'; B(H'^\dagger\sigma^a H')W'^a,$ $B'(H^\dagger\sigma^a H)W^a$ and their $\tilde{W}$ forms	bosonic, Hermitian	6	24	13/24	13/24	12/24; forbids $J_H J_H'$	12/24; forbids $J_H J_H'$
current- current	$J_f^\mu J_{g\mu},$ $f, g \in \{Q, u, d, L, e, H\},$ $(f, g) \neq (H, H); J_f^\mu = \bar{f}\gamma^\mu f$ (chiral f), $J_H^\mu = i[H^\dagger D^\mu H - (D^\mu H)^\dagger H]$	vector $\times$ vector, Hermitian	6	35	20/35	20/35	n/a	n/a
Yukawa portals	$(\bar{Q}u\tilde{H}) H' ^2, (\bar{Q}dH) H' ^2,$ $(\bar{L}eH) H' ^2$ and sector 1 $\leftrightarrow$ 2	scalar bilinear, complex	6	12	6/12	6/12	n/a	n/a
dipole portals	$B_{\mu\nu}(\bar{Q}'\sigma^{\mu\nu}u'\tilde{H}'),$ $B_{\mu\nu}(\bar{Q}'\sigma^{\mu\nu}d'H'),$ $B_{\mu\nu}(\bar{L}'\sigma^{\mu\nu}e'H')$ and sector 1 $\leftrightarrow$ 2	tensor bilinear, complex ($\tilde{B}$ forms dependent)	6	12	6/12	6/12	n/a	n/a
ν portal, dim. 5	$(LH)(L'H'),$ $(LH) \equiv \epsilon_{ij}L_iH_j$	mirror: $\overline{(LH)}(L'H');$ copy: $(LH)^T C(L'H');$ complex	5	2	1/2; phase fixed	2/2	n/a	n/a
ν portal, dim. 6	$(LH)\Gamma^\mu(L'D_\mu H')$ [($LD_\mu H$) $\Gamma^\mu(L'H')$ removed by IBP and EOM]	mirror: $(LH)^T C\gamma^\mu(L'D_\mu H');$ copy: $(LH)\gamma^\mu(L'D_\mu H');$ complex	6	2	0/2; forbids all	1/2; phase fixed	n/a	n/a
six-quark ($n-n'$)	$(udd)(u'd'd'), (udd)(Q'Q'd'),$ $(QQd)(u'd'd'),$ $(QQd)(Q'Q'd'), [SSS]$	mirror: $\overline{N_X}N_Y';$ copy: $N_X^T C N_Y';$ complex	9	10	5/10	8/10	n/a	n/a
$\Delta S = 2$	$J_f^\mu(sd)J_{g\mu}'(sd),$ $J_f^\mu(sd)J_{g\mu}'(ds), f, g \in \{Q, d\},$ $J_f^\mu(sd) = \bar{f}_s\gamma^\mu f_d$ (two flavors)	vector $\times$ vector, complex	6	16	10/16	10/16	n/a	n/a

TABLE II. Operators of the family: dim.-4 bosonic.

dim.-4 bosonic (3 real; survivors: $\mathcal{M}_P$ 2/3, $\tilde{\mathcal{M}}$ 3/3, $\mathcal{M}$ 1/3, $\tilde{\mathcal{M}}_P$ 2/3)

k	O_k	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$	$\mathcal{M}$	$\tilde{\mathcal{M}}_P$
1	$ H ^2 H' ^2$	1	$+O_1$	$+O_1$	$+O_1$	$+O_1$
2	$B_{\mu\nu}B'^{\mu\nu}$	1	$+O_2$	$+O_2$	$-O_2$	$-O_2$
3	$B_{\mu\nu}\tilde{B}'^{\mu\nu}$	1	$-O_3$	$+O_3$	$-O_3$	$+O_3$

TABLE III. Operators of the family: dim.-6 bosonic.
dim.-6 bosonic (24 real; survivors: $\mathcal{M}_P$ 13/24, $\tilde{\mathcal{M}}$ 13/24, $\mathcal{M}$ 12/24, $\tilde{\mathcal{M}}_P$ 12/24)

k	O_k	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$	$\mathcal{M}$	$\tilde{\mathcal{M}}_P$
1	$ H ^4 H' ^2$	1	$+O_2$	$+O_2$	$+O_2$	$+O_2$
2	$ H ^2 H' ^4$	1	$+O_1$	$+O_1$	$+O_1$	$+O_1$
3	$\partial_\mu H ^2 \partial^\mu H' ^2$	1	$+O_3$	$+O_3$	$+O_3$	$+O_3$
4	$J_H^\mu J'_{H\mu}$	1	$+O_4$	$+O_4$	$-O_4$	$-O_4$
5	$ H ^2 B'_{\mu\nu} B'^{\mu\nu}$	1	$+O_6$	$+O_6$	$+O_6$	$+O_6$
6	$ H' ^2 B_{\mu\nu} B^{\mu\nu}$	1	$+O_5$	$+O_5$	$+O_5$	$+O_5$
7	$ H ^2 B'_{\mu\nu} \tilde{B}'^{\mu\nu}$	1	$-O_8$	$+O_8$	$+O_8$	$-O_8$
8	$ H' ^2 B_{\mu\nu} \tilde{B}^{\mu\nu}$	1	$-O_7$	$+O_7$	$+O_7$	$-O_7$
9	$ H ^2 W'^a_{\mu\nu} W'^{a\mu\nu}$	1	$+O_{10}$	$+O_{10}$	$+O_{10}$	$+O_{10}$
10	$ H' ^2 W^a_{\mu\nu} W^{a\mu\nu}$	1	$+O_9$	$+O_9$	$+O_9$	$+O_9$
11	$ H ^2 W'^a_{\mu\nu} \tilde{W}'^{a\mu\nu}$	1	$-O_{12}$	$+O_{12}$	$+O_{12}$	$-O_{12}$
12	$ H' ^2 W^a_{\mu\nu} \tilde{W}^{a\mu\nu}$	1	$-O_{11}$	$+O_{11}$	$+O_{11}$	$-O_{11}$
13	$ H ^2 G'^A_{\mu\nu} G'^{A\mu\nu}$	1	$+O_{14}$	$+O_{14}$	$+O_{14}$	$+O_{14}$
14	$ H' ^2 G^A_{\mu\nu} G^{A\mu\nu}$	1	$+O_{13}$	$+O_{13}$	$+O_{13}$	$+O_{13}$
15	$ H ^2 G'^A_{\mu\nu} \tilde{G}'^{A\mu\nu}$	1	$-O_{16}$	$+O_{16}$	$+O_{16}$	$-O_{16}$
16	$ H' ^2 G^A_{\mu\nu} \tilde{G}^{A\mu\nu}$	1	$-O_{15}$	$+O_{15}$	$+O_{15}$	$-O_{15}$
17	$ H ^2 B_{\mu\nu} B'^{\mu\nu}$	1	$+O_{18}$	$+O_{18}$	$-O_{18}$	$-O_{18}$
18	$ H' ^2 B_{\mu\nu} B'^{\mu\nu}$	1	$+O_{17}$	$+O_{17}$	$-O_{17}$	$-O_{17}$
19	$ H ^2 B_{\mu\nu} \tilde{B}'^{\mu\nu}$	1	$-O_{20}$	$+O_{20}$	$-O_{20}$	$+O_{20}$
20	$ H' ^2 B_{\mu\nu} \tilde{B}'^{\mu\nu}$	1	$-O_{19}$	$+O_{19}$	$-O_{19}$	$+O_{19}$
21	$B_{\mu\nu} (H'^\dagger \sigma^a H') W'^{a\mu\nu}$	1	$+O_{22}$	$+O_{22}$	$-O_{22}$	$-O_{22}$
22	$B'_{\mu\nu} (H^\dagger \sigma^a H) W^{a\mu\nu}$	1	$+O_{21}$	$+O_{21}$	$-O_{21}$	$-O_{21}$
23	$B_{\mu\nu} (H'^\dagger \sigma^a H') \tilde{W}'^{a\mu\nu}$	1	$-O_{24}$	$+O_{24}$	$-O_{24}$	$+O_{24}$
24	$B'_{\mu\nu} (H^\dagger \sigma^a H) \tilde{W}^{a\mu\nu}$	1	$-O_{23}$	$+O_{23}$	$-O_{23}$	$+O_{23}$

TABLE IV. Operators of the family: current–current.
current–current (35 real; survivors: $\mathcal{M}_P$ 20/35, $\tilde{\mathcal{M}}$ 20/35)

k	O_k	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$
1	$J_Q^\mu J_{Q\mu}'$	1	$+O_1$	$+O_1$
2	$J_Q^\mu J_{u\mu}'$	1	$+O_7$	$+O_7$
3	$J_Q^\mu J_{d\mu}'$	1	$+O_{13}$	$+O_{13}$
4	$J_Q^\mu J_{L\mu}'$	1	$+O_{19}$	$+O_{19}$
5	$J_Q^\mu J_{e\mu}'$	1	$+O_{25}$	$+O_{25}$
6	$J_Q^\mu J_{H\mu}'$	1	$+O_{31}$	$+O_{31}$
7	$J_u^\mu J_{Q\mu}'$	1	$+O_2$	$+O_2$
8	$J_u^\mu J_{u\mu}'$	1	$+O_8$	$+O_8$
9	$J_u^\mu J_{d\mu}'$	1	$+O_{14}$	$+O_{14}$
10	$J_u^\mu J_{L\mu}'$	1	$+O_{20}$	$+O_{20}$
11	$J_u^\mu J_{e\mu}'$	1	$+O_{26}$	$+O_{26}$
12	$J_u^\mu J_{H\mu}'$	1	$+O_{32}$	$+O_{32}$
13	$J_d^\mu J_{Q\mu}'$	1	$+O_3$	$+O_3$
14	$J_d^\mu J_{u\mu}'$	1	$+O_9$	$+O_9$
15	$J_d^\mu J_{d\mu}'$	1	$+O_{15}$	$+O_{15}$
16	$J_d^\mu J_{L\mu}'$	1	$+O_{21}$	$+O_{21}$
17	$J_d^\mu J_{e\mu}'$	1	$+O_{27}$	$+O_{27}$
18	$J_d^\mu J_{H\mu}'$	1	$+O_{33}$	$+O_{33}$
19	$J_L^\mu J_{Q\mu}'$	1	$+O_4$	$+O_4$
20	$J_L^\mu J_{u\mu}'$	1	$+O_{10}$	$+O_{10}$
21	$J_L^\mu J_{d\mu}'$	1	$+O_{16}$	$+O_{16}$
22	$J_L^\mu J_{L\mu}'$	1	$+O_{22}$	$+O_{22}$
23	$J_L^\mu J_{e\mu}'$	1	$+O_{28}$	$+O_{28}$
24	$J_L^\mu J_{H\mu}'$	1	$+O_{34}$	$+O_{34}$
25	$J_e^\mu J_{Q\mu}'$	1	$+O_5$	$+O_5$
26	$J_e^\mu J_{u\mu}'$	1	$+O_{11}$	$+O_{11}$
27	$J_e^\mu J_{d\mu}'$	1	$+O_{17}$	$+O_{17}$
28	$J_e^\mu J_{L\mu}'$	1	$+O_{23}$	$+O_{23}$
29	$J_e^\mu J_{e\mu}'$	1	$+O_{29}$	$+O_{29}$
30	$J_e^\mu J_{H\mu}'$	1	$+O_{35}$	$+O_{35}$
31	$J_H^\mu J_{Q\mu}'$	1	$+O_6$	$+O_6$
32	$J_H^\mu J_{u\mu}'$	1	$+O_{12}$	$+O_{12}$
33	$J_H^\mu J_{d\mu}'$	1	$+O_{18}$	$+O_{18}$
34	$J_H^\mu J_{L\mu}'$	1	$+O_{24}$	$+O_{24}$
35	$J_H^\mu J_{e\mu}'$	1	$+O_{30}$	$+O_{30}$

TABLE V. Operators of the family: Yukawa portals.
Yukawa portals (12 real; survivors: $\mathcal{M}_P$ 6/12, $\tilde{\mathcal{M}}$ 6/12)

k	O_k	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$
1	$(\bar{Q} u \tilde{H}) H' ^2$	2	$+O_2$	$+O_2$
2	$(\bar{Q}' u' \tilde{H}') H ^2$	2	$+O_1$	$+O_1$
3	$(\bar{Q} d H) H' ^2$	2	$+O_4$	$+O_4$
4	$(\bar{Q}' d' H') H ^2$	2	$+O_3$	$+O_3$
5	$(\bar{L} e H) H' ^2$	2	$+O_6$	$+O_6$
6	$(\bar{L}' e' H') H ^2$	2	$+O_5$	$+O_5$

TABLE VI. Operators of the family: dipole portals.
dipole portals (12 real; survivors: $\mathcal{M}_P$ 6/12, $\tilde{\mathcal{M}}$ 6/12)

k	O_k	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$
1	$B_{\mu\nu}(\bar{Q}'\sigma^{\mu\nu}u'\tilde{H}')$	2	$+O_2$	$+O_2$
2	$B'_{\mu\nu}(\bar{Q}\sigma^{\mu\nu}u\tilde{H})$	2	$+O_1$	$+O_1$
3	$B_{\mu\nu}(\bar{Q}'\sigma^{\mu\nu}d'H')$	2	$+O_4$	$+O_4$
4	$B'_{\mu\nu}(\bar{Q}\sigma^{\mu\nu}dH)$	2	$+O_3$	$+O_3$
5	$B_{\mu\nu}(\bar{L}'\sigma^{\mu\nu}e'H')$	2	$+O_6$	$+O_6$
6	$B'_{\mu\nu}(\bar{L}\sigma^{\mu\nu}eH)$	2	$+O_5$	$+O_5$

TABLE VII. Operators of the family: ν portal, dim. 5.
 ν portal, dim. 5 (2 real; survivors: $\mathcal{M}_P$ 1/2, $\tilde{\mathcal{M}}$ 2/2)

k	O_k (mirror content)	O_k (copy content)	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$
1	$\overline{(LH)}(L'H')$	$(LH)^T C(L'H')$	2	$+O_1^\dagger$	$+O_1$

TABLE VIII. Operators of the family: ν portal, dim. 6.
 ν portal, dim. 6 (2 real; survivors: $\mathcal{M}_P$ 0/2, $\tilde{\mathcal{M}}$ 1/2)

k	O_k (mirror content)	O_k (copy content)	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$
1	$(LH)^T C\gamma^\mu(L'D_\mu H')$	$\overline{(LH)}\gamma^\mu(L'D_\mu H')$	2	$-O_1$	$-O_1^\dagger$

TABLE IX. Operators of the family: six-quark (n - n').
six-quark (n - n') (10 real; survivors: $\mathcal{M}_P$ 5/10, $\tilde{\mathcal{M}}$ 8/10)

k	O_k (mirror content)	O_k (copy content)	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$
1	$\bar{N}_{udd}N'_{udd}$	$N_{udd}^T C N'_{udd}$	2	$+O_1^\dagger$	$+O_1$
2	$\bar{N}_{udd}N'_{QQd}$	$N_{udd}^T C N'_{QQd}$	2	$+O_3^\dagger$	$+O_3$
3	$\bar{N}_{QQd}N'_{udd}$	$N_{QQd}^T C N'_{udd}$	2	$+O_2^\dagger$	$+O_2$
4	$\bar{N}_{QQd}N'_{QQd}$	$N_{QQd}^T C N'_{QQd}$	2	$+O_4^\dagger$	$+O_4$
5	$\mathcal{O}_{[SSS]}$	$\mathcal{O}_{[SSS]}$	2	$+O_5^\dagger$	$+O_5$

TABLE X. Operators of the family: $\Delta S = 2$.
 $\Delta S = 2$ (16 real; survivors: $\mathcal{M}_P$ 10/16, $\tilde{\mathcal{M}}$ 10/16)

k	O_k	real	$\mathcal{M}_P$	$\tilde{\mathcal{M}}$
1	$J_Q^\mu(sd)J'_{Q\mu}(sd)$	2	$+O_1$	$+O_1$
2	$J_Q^\mu(sd)J'_{Q\mu}(ds)$	2	$+O_2^\dagger$	$+O_2^\dagger$
3	$J_Q^\mu(sd)J'_{d\mu}(sd)$	2	$+O_5$	$+O_5$
4	$J_Q^\mu(sd)J'_{d\mu}(ds)$	2	$+O_6^\dagger$	$+O_6^\dagger$
5	$J_d^\mu(sd)J'_{Q\mu}(sd)$	2	$+O_3$	$+O_3$
6	$J_d^\mu(sd)J'_{Q\mu}(ds)$	2	$+O_4^\dagger$	$+O_4^\dagger$
7	$J_d^\mu(sd)J'_{d\mu}(sd)$	2	$+O_7$	$+O_7$
8	$J_d^\mu(sd)J'_{d\mu}(ds)$	2	$+O_8^\dagger$	$+O_8^\dagger$